Everyday police officers use radars to spot speeders on the road. As the car approaches, police just shoot the radar and get a number for the speed. But, why does it work? By using angles and trigonometry functions, this radar can come out with accurate speeds for cars. However, under certain condition, these radars can be very unbeneficial. In this project, we will find factors that affect the accuracy of using radars to measure the speed of the oncoming car.

1. Suppose an officer in a car 15 feet off the side of the road (point B in Figure 1). A vehicle approaches traveling 70 mph (point A figure 1). We want to calculate the speed of the car reported by the radar unit when the car is 100 feet away.

   Figure 1
   
   a. We need to compute the speed of the car by measuring the difference in the length of BA and the length of BC. Generally, the length of AC (the true distance traveled by the car in \( t \) seconds) is not the same as the difference in the lengths of BA and BC (the distance the radar gun uses to compute the speed of the car). Where would the police officer need to be positioned for the distance the car travels and the distance the radar gun measures to be exactly the same?

   b. We need to compute the length of BC as the first step in obtaining a function for computing the speed reported by the radar gun. To do so be use the formula
   
   \[ a = \sqrt{b^2 + c^2 - 2bc\cos A}, \]
   
   where \( a \) is the length of BC, \( b \) is the length of AC, and \( c \) is the length of BA.

   i. Explain why equation (1) is valid.
   
   The equation is valid because of the Law of Cosines: The Square of one side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

   ii. Find the \( \cos A \)

   \( 
   \cos A = \frac{\text{adjacent}(A)}{\text{hypotenuse}(H)}. \]

   We know that \( H = c \), which=100ft. However, we do not know the whole length of \( A \). This is where Pythagoreans theorem comes in. We know the police officer is standing 15ft from the road and 100ft from the car. Therefore, our equation will
\[ \text{be} 100^2 = 15^2 + A^2. \text{ From there you can solve for } A = \sqrt{(100^2 + 15^2)}. \]

Final answer for \( A \) is \( \sqrt{9775} \). Now we can finally answer the \( \cos A \), which now equals \( \sqrt{9775 \text{ ft}} \) \( \div 100 \text{ ft} \).

\( \text{iii.} \) Determine an expression for \( b \) in terms of rate and time with the rate in feet per second and time as a variable in seconds. Remember that the car is traveling at 70 mph.

\[ \text{In order to find an expression for } b, \text{ you must know that} \]
\[ \text{Distance} = \text{Rate} \times \text{Time}. \text{ Once you know this, then } b \text{ can be subbed in for distance}
\]
\[ \text{because } b \text{ is a distance. The last step is to change 70 mph to ft/sec, which is } 308/3 \text{ ft/sec. Now we have an equation: } b = \frac{308}{3} \times t. \]

\( \text{iv.} \) Substitute your expression for \( b \) and \( \cos A \) into the equation (1). You should now have a function where your input, \( t \), is in seconds and your output, \( a \) is in feet.

\[ \text{Equation (1)} \]
\[ a = \sqrt{b^2 + c^2 - 2b\cos A}. \text{ Now where ever } b \text{ is sub in} \]
\[ \frac{308}{3} \times t. \]

Final equation: \[ a = \sqrt{\frac{308}{3} \times t + 100^2 - 2\left(\frac{308}{3} \times t\right)(100)(\sqrt{9775})}. \]

\( \text{c.} \) The speed reported by the radar gun is
\[ r_{\text{rep}} = \frac{100 - a}{t}. \]

Find a function where your input, \( t \), is in seconds and your output, \( r_{\text{rep}} \), is in feet per second.

\( \text{d.} \) Use the list of times given in the following table to compute various values of \( r_{\text{rep}} \).
Report your answers in both feet per second and miles per hour. Round to four decimal places.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>( r_{\text{rep}} ) (ft/sec)</th>
<th>( r_{\text{rep}} ) (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>101.3731</td>
<td>69.1180</td>
</tr>
<tr>
<td>0.01</td>
<td>101.4931</td>
<td>69.1998</td>
</tr>
<tr>
<td>0.0001</td>
<td>101.5049</td>
<td>69.2079</td>
</tr>
<tr>
<td>0.000001</td>
<td>101.5050</td>
<td>69.2080</td>
</tr>
</tbody>
</table>

\( \text{2.} \) In the previous table, the time becomes progressively smaller. Let’s now assume the time is instantaneous. In this case, it can be shown (using calculus) that \( r_{\text{rep}} = r_{\text{car}} \cos A \). Use this formula to compute \( r_{\text{rep}} \) for question 1. How does this answer compare with the answers you obtained in part (d)?
For the rest of this project, use the formula

\[ r_{rep} = r_{car} \cos A \]

to compute the speed reported by the radar gun.

\[ R_{car} = 70 \text{mph}, \cos A = \frac{\sqrt{9775}}{100} \]

By subbing in these numbers, \( R_{rep} \) should equal 69.2080mph. This means that the radar is reading 69.2080mph for the car when the car is actual going 70mph.

3. An officer is in a car 15 ft off the side of the road. A vehicle approaches traveling 70mph. What will the radar read as the car’s speed when the car is 50 feet away?

Now instead of having the \( \cos A \) refer to the hypotenuse being 100ft it will now refer to 50ft. Therefore, the equation will be \( R_{rep} = 70 \times \frac{\sqrt{2275}}{50} \). Now the radar reads 66.7757mph.

4. Compare your answers in questions 2 and 3. Notice that the accuracy of the measurement (i.e. \( |r_{car} - r_{rep}| \)) changed as the vehicle came closer. Let’s determine how the accuracy of the radar is affected by changing its distance from the vehicle, the speed of the vehicle, and the distance of the police officer from the side of the road.

a. First, let’s look at this problem graphically.

i. Holding speed of the car constant at 70 mph and distance from the police car to the side of the road constant at 25 ft, find a function where the input is the distance from the police car to the speeding car and the output is the reported speed. Graph this function.
ii. Holding distance from the speeding car constant at 100 ft and distance from the side of the road constant at 25 ft, find a function where the input is the speed of the car and the output is the speed reported by the radar unit. Graph this function.

\[ x = \frac{\sqrt{100^2 - 25^2}}{100} \quad \text{X} = \text{Speed of car} \]

![Graph of Speed of Radar vs. Speed of Car](image)

iii. Holding distance from the car constant at 100 ft and speed of the car constant at 70 mph, find a function where the input is the distance from the side of the road and the output is the speed reported by the radar unit. Graph this function.

\[ 70 \times \frac{\sqrt{100^2 - x^2}}{100} \quad \text{X} = \text{Distance of police officer from road} \]

![Graph of Radar Speed vs. Police Officers Distance](image)
b. Using your graphs from part (a) as a guide, determine what happens to the accuracy of the radar reading as the vehicle comes closer. Determine what happens to the accuracy of the radar reading as the target vehicle goes faster. Determine what happens to the accuracy of the radar reading as the police officer sits farther off side of the road. Justify each of your three explanations with either a symbolic or a geometric argument.

Look at the graphs above, as the car approaches closer to the police officer, the less accurate the radar is. As the car’s speed increases, the less accurate the radar is. As the police officer steps further from the road, the less accurate the radar is. Therefore, if the police officer is as close as he can to the road, the car is going slow, and distance from the car and the police officer is as far as the radar can read, that would be the most accurate reading.

5. In a realistic situation, people often first decide on how much error they are willing to have and then change the other variables accordingly. For example, the police officer may want the reading of the radar unit to be within 1 mph of the actual speed of the car. Assume the police officers are sitting in their car 10 feet off the side of an approaching car traveling 75 mph. How far away should the car be when the police officers use their radar unit to ensure that their reading is within the desired accuracy range?

In order to be 1 mph within the actual speed of the approaching car, the new equation for the radar speed is \(74 = 75 * \sqrt{x^2 - 10^2} / x\). Next, look at the graph and where it hits 74mph, that will be the distance where the police should shoot the radar to get the 1mph difference. In this graph below, the distance where the police should shoot is between 60-65ft.
6. A person who receives a speeding ticket for going 57mph in a 45 mph zone contests the
ticket on the grounds that, because of the angle effect, his speed was actually less than
what the radar indicated. As the court’s expert witness (on the law of cosines), write a
response to the judge indicating why you think the ticketed driver should or should not
be fined for speeding.

Dear Judge,

I am writing to you in response to the speeding ticket trial. With the previous
information above, I would like to inform you that radars do not calculate speeds above
the car's original speed. In fact, radars do the opposite. They calculate speeds under the
car's original speed (check graphs and calculations above). Therefore, the person who
received this ticket is actually guilty.

Thank you

Sincerely,

Courtney Dye